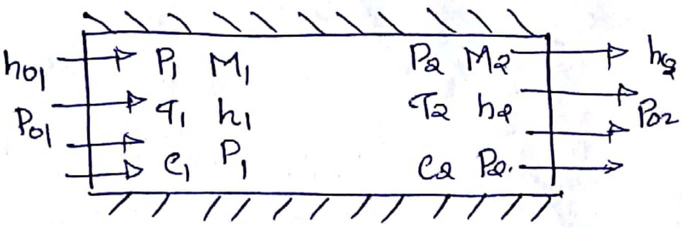
Fanno Flow

* Also known as simple friction flow.
* Flow in a constant area duct with friction in absence of work and heat transfer
* Changes in flow parameter mainly due to friction.
* Application : ducts of reasonably short length can be modelled as Fanno flow. like in aerospace propulsion system, transport of fluids in chemical process plant, thermal & nuclear power plants

Assumptions for studying Fanno Flow

1) Constant area duct flow
2) No heat transfer, hence flow can be approximated as adiabatic
3) No work transfer
4) Perfect gas
5) One dimensional, steady flow
6) Body forces are negligible.
7) Wall friction is the sole driving potential.



Governing Equations

1) Continuity Equation

$$\dot{m} = \rho_1 A C_1 = \rho_2 A C_2$$

$$\dot{m}/A = \rho_1 C_1 = \rho_2 C_2 = G \quad \text{(mass flux/ mass velocity)}$$

2) <u>Momentum Equation</u> ②

In fanno flow friction due to viscosity is prominent and cannot be neglected. Hence the net external force will be sum of pressure force and shear force

$$-A\,dp - \tau_w\,dA_w = \dot{m}\,dc.$$

τ_w - shear stress.
dA_w - wetted wall area.

$$-A\,dp - 4f\frac{dx}{D}\frac{\rho c^2}{2} = \rho A\,dc.$$

3) <u>Energy Equation</u>

$$h_0 = h_1 + \frac{c_1^2}{2} = h_2 + \frac{c_2^2}{2}$$

4) <u>Equation of state</u>

$$s = s(\rho, h)$$

$$P = \rho R T.$$

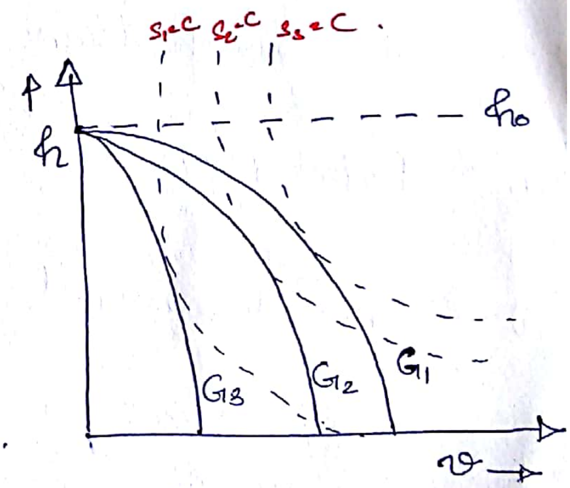
<u>Fanno line</u>

$$h_0 = h + \frac{c^2}{2} = \text{Constant}$$

$$G = \rho c.$$

$$c^2 = \frac{G^2}{\rho^2}$$

$$h_0 = h + \frac{G^2}{2\rho^2} = \text{Constant}.$$



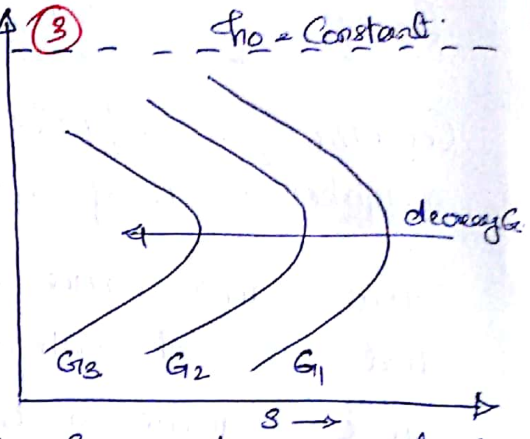
$G_1 < G_2 < G_3$

For a given value of 'h_0' & 'G' ~~values~~ we have set of values for 'h' & 's' during a fanno flow.

The curves obtained by plotting these h s values for different values of G and constant value of h_0 is shown in graph above.

These curves represent fanno flow for constant h_0 & given values of G and are called Fannolines. The equation is called Fannoline Equation

From equation of state $\rho = f(s, h)$



$$h_0 = h + \frac{G^2}{2\rho^2}$$

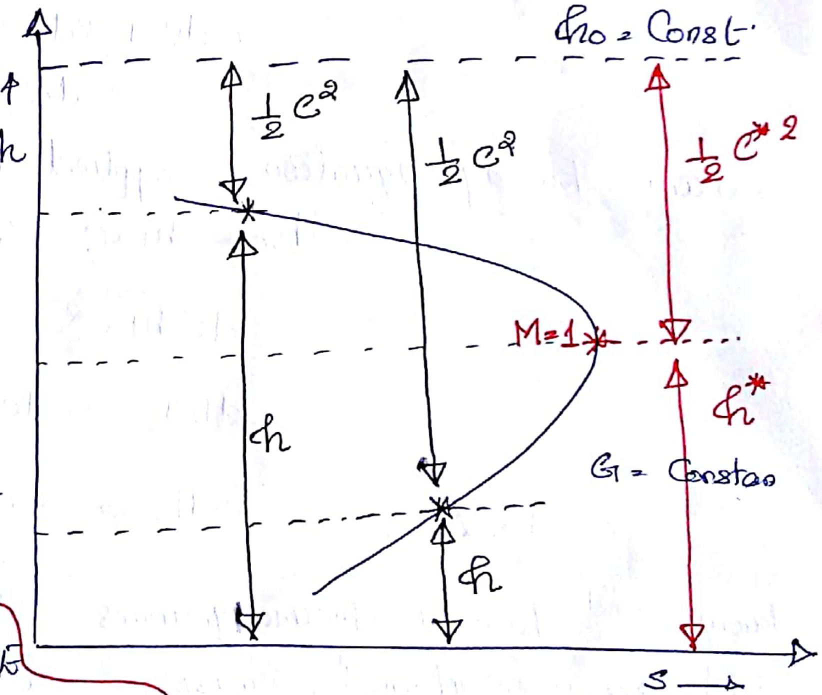
$$h_0 = h + \frac{G^2}{2\left[f(s,h)\right]^2}$$

* Using this equation, for constant value of h_0 and given values of G we can obtain curves representing Fanno Flow (Fanno lines) in h-s diagram (Mollier diagram)

*. It is seen that when mass flow rate increases, fanno line shifts towards the left side.

* The shape of curve is such that there is a point of maximum entropy and its ~~steps~~ splits the curve into two portions.

Consider a point lying on the upper section of curve. If we compare the K.E. ($\frac{1}{2}c^2$) between this point and at that of maximum entropy we can see that velocity at this point is less than that of sound.



HENCE UPPER SECTION OF CURVE REPRESENTS SUBSONIC FANNO FLOW

Similarly comparing K.E. ($\frac{1}{2}c^2$) of a point in lower region with maximum entropy condition we see that velocity at this point is greater than velocity of sound

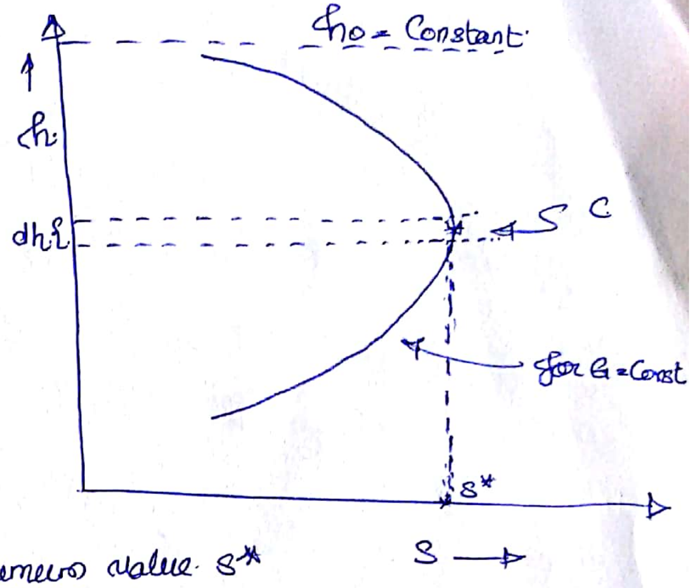
HENCE LOWER SECTION OF CURVE REPRESENTS SUPERSONIC FANNO Flow

Condition for Maximum Entropy ④ (very important)

Consider an infinitesimal change of state taking place at the neighbourhood of maximum entropy state (C).

From Fanno curve we can notice that for ~~st~~ such a small change at C, change in enthalpy 'dh' occurs at constant ~~enth~~ entropy.

Hence the change in entropy in this infinite-simal process can be considered zero i.e.,



$$ds = 0$$

$$s = \text{Constant} = \text{Maximum value } s^*$$

From continuity equation applied for Fanno flow

$$G = \rho c = \text{Constant}$$

$$d(\rho c) = 0$$

$$c\,d\rho + \rho\,dc = 0$$

$$dc = -c\frac{d\rho}{\rho} \qquad \text{—— (1)}$$

From Energy equation applied to Fanno flow..

$$h_0 = h + \frac{c^2}{2} = \text{Constant}$$

$$d\left(h + \frac{c^2}{2}\right) = 0$$

$$dh + \frac{2c\,dc}{2} = 0$$

$$dh = -c\,dc \qquad \text{—— (2)}$$

From 2nd Law of thermodynamics $T ds = dh - \frac{dp}{\rho}$

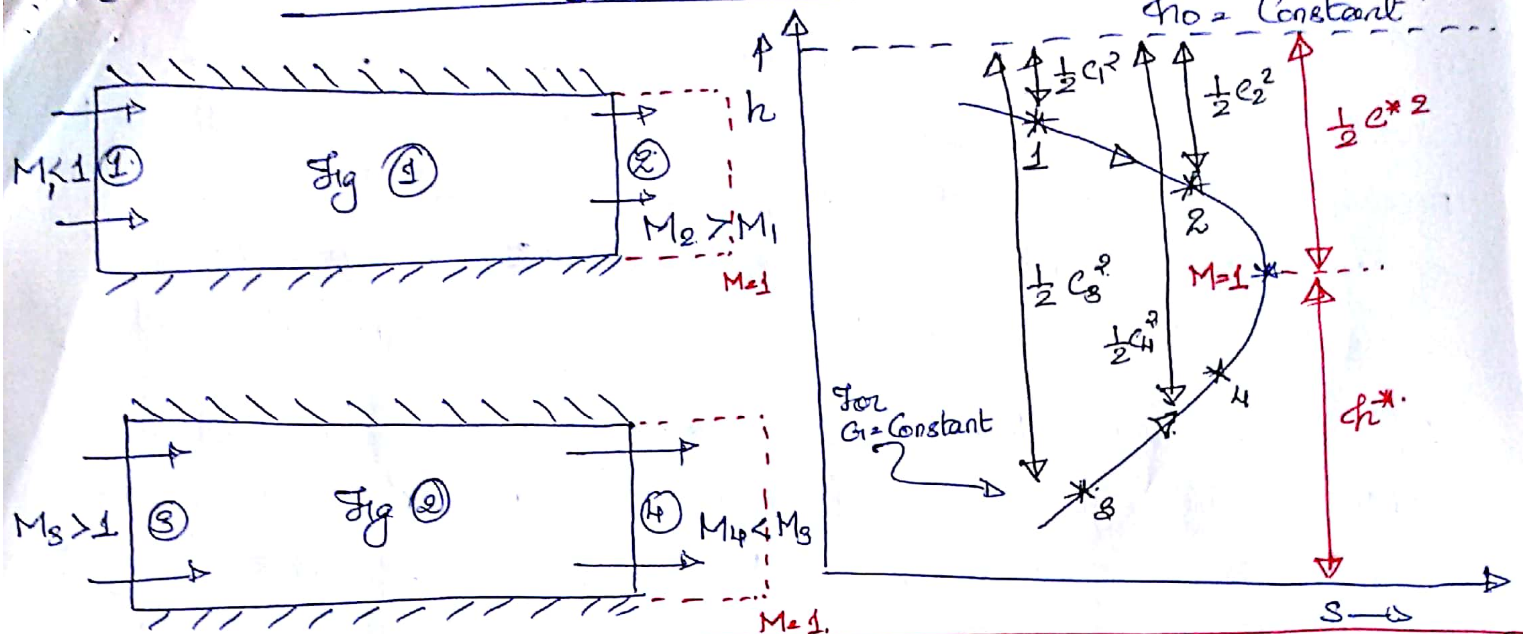
In this infinitesimal change $ds = 0 \Rightarrow dh = \frac{dp}{\rho}$ —— (3)

Substitute (1) in (2) for 'dc' $\Rightarrow dh = c^2\frac{d\rho}{\rho}$ —— (4)

Equating (3) & (4) $\frac{dp}{\rho} = c^2\frac{d\rho}{\rho} \Rightarrow c^2 = \frac{dp}{d\rho}$

Here entropy is constant hence the infinitesimal change can be considered isentropic. Therefore $c^2 = (dP/d\rho)_s \Rightarrow c^2/a^2 = 1 \Rightarrow M^2 = 1 \Rightarrow \underline{M = 1}$

Effect of friction on Mach number. 5



FOR FANNO FLOW FRICTION RESULTS IN IRREVERSIBILITY & ENTROPY INCREASE

Consider a subsonic fanno flow with inlet condition ①. When the flow proceeds to reach condition ② it can move only towards right side of fanno line which is in direction of increasing entropy. (moving left means entropy decreases which is not the case for friction flow hence violates 2nd law of thermodynamics).
Therefore the inlet & exit of are presented by points 1 & 2. has shown in h-s diagram. Comparing the two points we see that pressure, temperature and density decreases and flow velocity and Mach number increases.

HENCE FRICTION CAUSES SUBSONIC FLOW TO ACCELERATE

Consider a supersonic fanno flow with inlet conditions ③. When the flow proceeds to reach condition ④ satisfying the 2nd law of thermo dynamics the points ③ & ④ should be represented ~~by~~ as shown in h-s diagram. Comparing ③ & ④ we observe that pressure, temperature and density increases and Mach number, fluid velocity decreases.

HENCE FRICTION CAUSES SUPERSONIC FLOW TO DECELERATE

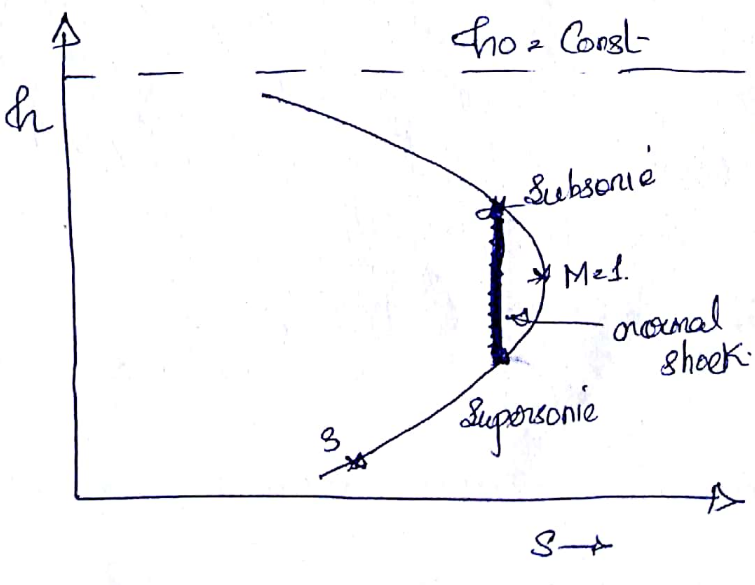
It can also be observed that a subsonic under action of friction can accelerate till Mach number becomes unity. Further the flow cannot be accelerated to supersonic value because for that the position will shift towards left in Fanno line. This is violation of 2nd law as entropy becomes negative. Hence maximum acceleration corresponds to Mach number unity. (shown by dashed line)

If in a duct flow becomes sonic in between and there is still length left to reach exit, the properties upstream of flow changes such that Mach number at inlet will get reduced. This causes a reduction in mass flux and the entire curve (fanno line) shifts towards right in h-s diagram. New inlet Mach number will be such that by the time flow reaches exit M=1

The similar observation can be made in supersonic section. A supersonic flow, under the influence of friction will decelerate till Mach number becomes unity. Going beyonds is again violation of 2nd law. (shown by dashed line)

If in a duct flow decelerate and reaches M=1 and the pipe length still exists, then normal shock occurs converting the supersonic flow to subsonic instantly. The occurance of shock depends on back pressure into which flow has to exit and friction factor.

When Mach number becomes unity under the influence of friction flow is said to be choked & is referred to as FRICTIONAL CHOKING.

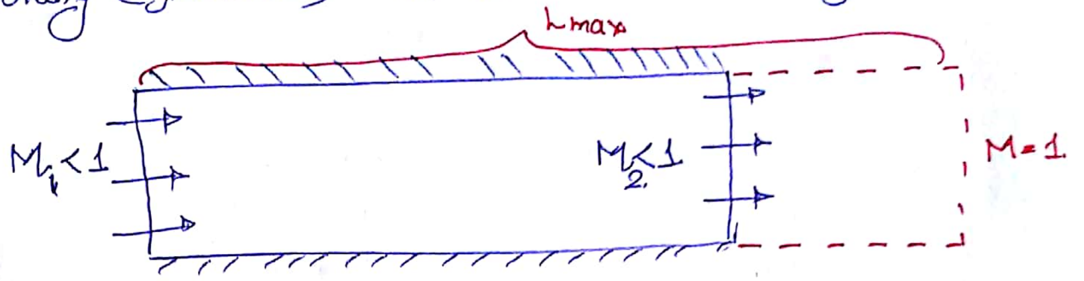


Critical length of duct (L_{max})

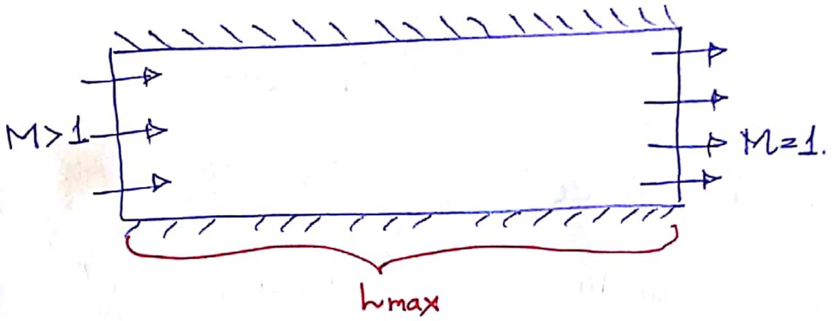
(7)

For a given inlet condition, the definite length of duct which can cause choking (frictional) is called critical length.

Flow with $M=1$ not lying within this duct.



Flow with $M=1$. lying within this duct.



Using this we can find out duct length. i.e..

duct length $L = L_{max\,(\text{inlet Mach no})} - L_{max\,(\text{exit Mach no})}$

Frictional coefficient (f)

defined as ratio of wall shearing stress to dynamic head of stream

$$f = \frac{\tau_w}{\frac{1}{2}\rho c^2}$$

To obtain the friction coefficient over a length (average friction factor)

$$\bar{f} = \frac{1}{L}\int_0^L f\,dx.$$

To obtain over critical length

$$\bar{f} = \frac{1}{L_{max}}\int_0^{L_{max}} f\,dx.$$

$$\frac{4f\,dx}{D} = \frac{1-\gamma M^2}{\gamma M^4}\,dM^2$$

$$\int_0^{L_{max}} \frac{4f\,dx}{D} = \int_M^{M=1} \frac{1-\gamma M^2}{\gamma M^2}\,dM^2.$$

To obtain relation connecting Mach number & L_{max}

we get {Eq 6.9, Page 8, Gas tables}

Property Variation due to effect of Friction

Property	Subsonic (M<1)	Supersonic (M>1)
pressure	decreases	Increases
Temperature	decreases	Increases
density	decreases	Increases
Stagnation Pressure	decreases	decreases
Impulse Function	decreases	decreases
Velocity	increases	decreases
Mach Number	increases	decreases.

Isothermal Flow

* Flow through constant area duct of very large length with friction and heat transfer is referred to as Isothermal flow.

* Flow driving potentials – Friction & heat transfer.

* It doesn't comes under classification of simple flow 'coz 2 driving potential.

* Static temperature can be approximated has constant throughout the flow.

* When temperature remains constant, frictional coefficient also remains constant.

* "A compressible flow with low Mach number in constant area which is in close contact with a constant temperature environment" In such a condition there will be sufficient time available for heat transfer to occur and temperature can be maintained constant.
  Eg: Natural gas pipelines

Assumptions

1) Constant area duct.
2) no body forces
3) no work transfer
4) 1-D, steady flow
5) Frictional flow with constant temp.
6) Perfect gas.

Governing Equations

1) Continuity Equation
$$\rho c = \text{Constant}$$

2) $$-A\,dA - \frac{4f\,dx}{D}\,\frac{\rho c^2}{2} = \rho A c\,dc.$$

3) $$Q = h_{02} - h_{01}.$$

4) $$p = f(h, s)$$
$$\rho = f(h, s)$$

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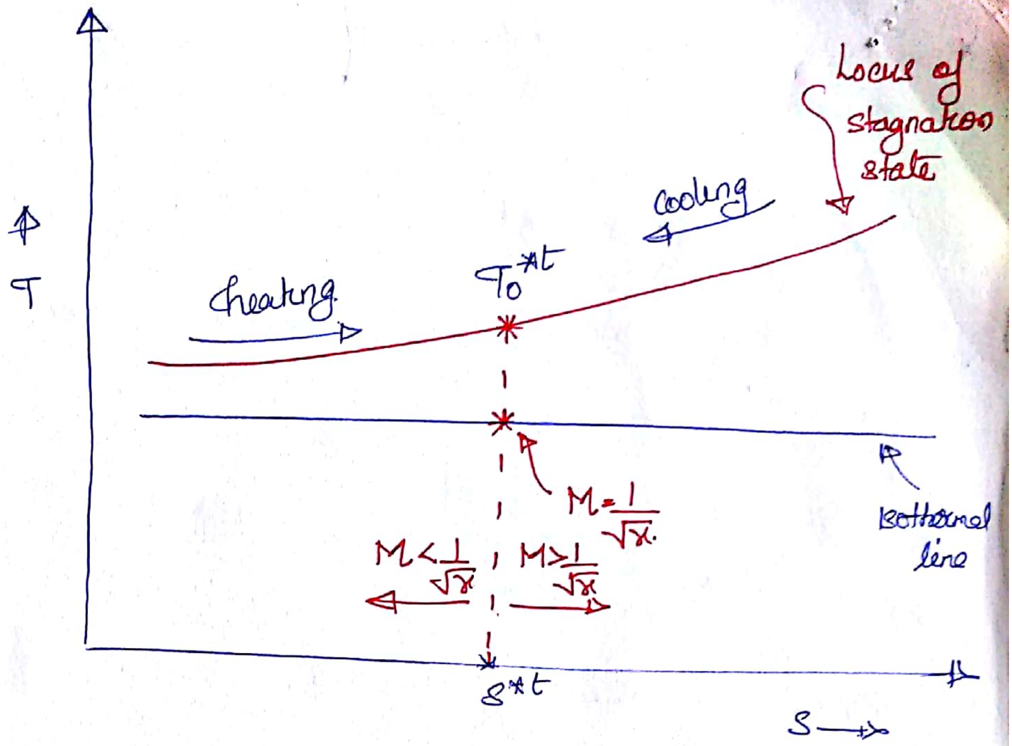
Mollier Diagram for Isothermal Flow



LIMITING VALUE OF MACH NUMBER

$$M = \frac{1}{\sqrt{\gamma}}$$

$$dT = 0$$

$$h_{01} \neq h_{02}$$

$$T_{01} \neq T_{02}$$

$$P_{01} \neq P_{02}$$

Critical condition represented by "$*t$".

Critical length, length of duct to be covered to reach $M = \frac{1}{\sqrt{\gamma}}$